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Distributed adaptive tracking backstepping control in networked nonidentical Lagrange systems

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Abstract In this paper, the problem of adaptive tracking control in networked nonidentical Lagrange systems is investigated via backstepping schemes. Two distributed tracking control algorithms are designed for directed network topology graph with a spanning tree, where both the leader's position and its velocity are assumed to be varying. Some generic criteria for adaptive tracking control algorithms with uncertain external disturbance and parametric uncertainties are presented. It is shown that the proposed algorithms only require a subset of the followers access to leader's position. Furthermore, the adaptive control algorithm for uncertain external disturbance systems is robust, and it can avoid online measurement for neighbors' velocities and efficiently eliminate the chattering during tracking. The results show that all followers can track the leader's dynamics. Two examples and their simulations show the effectiveness of the proposed algorithms.

Keywords Tracking backstepping control · Distributed adaptive algorithm · Networked Lagrange systems · Nonidentical · Parametric uncertainties

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1 Introduction

As is well known, Lagrange dynamics represent a class of mechanical systems including autonomous vehicles, robotic manipulators, and walking robots. Dynamics and control of Lagrange systems have been a significant topic over the past decades to both the scientific and the engineering communities. A wide variety of control strategies have been proposed and investigated for Lagrange systems which include linear and nonlinear feedback control [1], PD-type control [1,2], time-delay feedback control [3], and the open-loop optimization control [4–6], among many others. In recent years, coordinated control of networked Lagrange systems has attracted increasing attention from various fields of science and engineering. This problem arises in many application domains, including the control of multiple robot manipulators, formation control of UAVs, and mobile sensor networks [3,7–15], but its dynamics analysis is still very challenging due to the inherent nonlinearity and strong coupling between its generalized coordinates. Among the existing researches, the synchronization issue of Lagrange network systems possessing a nonlinear inertia matrix is much more involved and difficult. Ren [12] presented the distributed leaderless consensus algorithms for networked Lagrange systems under an undirected graph, Mabrouk [13] considered the global output tracking control for a class of Euler-Lagrange systems via a dynamic control law in the framework of triangular form, Wu and Zhou [8,9] proposed an analysis procedure for impulsive synchronization motion in networked open-loop multi-body systems formulated by Lagrange dynamics. In addition Mei, Ren, and Ma [16] further put forward to the distributed coordinated tracking schemes for undirected networked Lagrange systems with parametric uncertainties.

A large literature is available on the problem of synchronization of Lagrange networks with identical node's dynamics [8]; however, in some practical applications it is desirable to synchronize nonidentical Lagrange systems, where all agents can have different dynamics, and even dynamics uncertainties occur in some case because of a wide variety of environmental factors including the impact of various constraints, the imprecision of parameter measurements, and external disturbances [7,16]. These uncertainties will inevitably degrade the performances of networked controlled systems. As a result, some authors have recently devoted to propose different kinds of adaptive distributed algorithms for synchronization of networked Lagrange systems from various perspectives [16–19]. For example, Nuño, et al. [17] introduced an adaptive controlled synchronization algorithm for nonidentical Lagrange network with parametric uncertainties; Chung and Slotine [18] studied the cooperative robot concurrent synchronization of nonidentical Lagrange systems; the proposed decentralized strategy is further extended to adaptive synchronization for parametric uncertain robot models. Chen and Lewis [19,20] presented the distributed adaptive tracking algorithms of the unknown networked Lagrange systems, and the tracking errors are uniformly ultimately bounded.

On the other hand, the backstepping approach is viewed as a systematic scheme following a step-by-step algorithm based on the construction of feedback control law and Lyapunov functions. The advantages of the backstepping approach include the following three aspects: First, it has the flexibility to avoid cancelations of useful nonlinearities and achieve regulation and tracking properties. Second, this method is based on the redesign of the Lyapunov function and careful gain selections, and the controller design process can be a simple routine work if a proper Lyapunov candidate function is chosen. Finally, it can guarantee asymptotic stability, and it also has robustness to some unmatched uncertainties [21–23]. The importance of the backstepping control method lies in that it can well prescribe the practical structures of the designed controller, and so it can be usually used as an effective control strategy to stabilize and synchronize underlying dynamical systems in some practical applications [23–27]. Recently, the backstepping methodology has been developed for the design and analysis of adaptive synchronization algorithms for networked Lagrange systems [26,27] and references therein.

With the aforementioned background, in this paper, by the combination of backstepping approach and directed graph theory, we are mainly interested in adaptive tracking control of networked nonidentical Lagrange systems under the assumptions that the graph contains a spanning tree, and only a subset of the followers have access to the leader's position; the proposed algorithms work to track time-varying position and velocity vector of the leader. In addition, the benefits of the proposed algorithms can be summarized as the following three points: To begin with, compared with the tracking or synchronization Lagrange systems with undirected graph [8,9,12,16,27], the proposed techniques can deal with tracking control under general directed graph with a spanning tree. Moreover, in contrast to the tracking control algorithms with measuring the neighbors' both positions and velocities [9,17,19], the proposed control algorithm for uncertain external disturbance systems is only positions measurement of the neighbors. Lastly, different from the adaptive algorithms [12,16,21,23,26], which may cause chattering generated by the sign function, the proposed adaptive tracking algorithm with uncertain external disturbance is robust, and it can efficiently eliminate the chattering during tracking.

The outline of the paper is organized as follows: in Sect. 2, notations, properties, and assumptions of networked Lagrange systems are shown; in Sect. 3, adaptive tracking control algorithms for nonidentical Lagrange systems with parametric uncertainties and external disturbance are presented; in Sect. 4, two application examples and simulations are demonstrated and validated the results; in Sect. 5, we will draw some conclusions.

2 Preliminary and problem statement

Let $\mathbb{R} = (-\infty, +\infty)$ be the set of real numbers, and $\mathbb{N} = \{1, 2, \cdots\}$ be the set of nonnegative integer numbers. For the vector $u \in \mathbb{R}^n$, u^T denotes its transpose. The norm of the vector u is defined as $\|u\| = \sqrt{u^T u}$. $\mathbb{R}^{n\times n}$ stands for $n \times n$ the set of real matrices. $A \otimes B$ denotes the Kronecker product of matrix A and B. $A \circ B$ denotes the Hadamard product of matrix A and B. Assume A is symmetric matrix, $A < 0$, (> 0) denotes negative (positive) definite matrix.

Consider a directed graph $\mathbb{G} = (V, E, A)$ of order N $(N \geq 2)$ with a set of nodes $V = \{1, 2, \cdots, N\}$, a set of edges $E \subset V \times V$, and a adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ with nonnegative adjacency elements a_{ij}. The edge (i, j) in the edge set of a directed graph denotes that agent j can obtain information from agent i, but not necessarily vice versa. A is defined as $a_{ij} > 0$ if $(j, i) \in E$, otherwise $a_{ij} = 0$ for all $i \neq j$, and $a_{ii} = 0$ for all $i \in V$. Let Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ associated with A be defined as $l_{ii} = \sum_{k=1,k\neq i}^{N} a_{ik}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

Suppose that there exist N followers, labeled as agents 1 to N, and a leader labeled 0 as $(N + 1)$th agent, the directed graph denotes as $\overline{\mathbb{G}}$, the Laplacian matrix associated with $\overline{\mathbb{G}}$ denotes as $\overline{L}$. Let $q_0(t) \in \mathbb{R}^n$ and $\dot{q}_0(t) \in \mathbb{R}^n$ denote the leader's vector of generalized coordinates and velocity, respectively. The graph $\overline{\mathbb{G}}$ describes directed position communication among agents. Let $H = L + B$, where $B = \mathrm{diag}(b_{10}, b_{20}, \cdots, b_{N0})$, $b_{i0} > 0$ if the follower i has access to q_0 and $b_{i0} = 0$ if otherwise [14,15,19,20].

The n-degree Euler-Lagrange equations for the N followers are described as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i,\quad i = 1, 2, \cdots, N, \tag{1}$$

where $q_i \in \mathbb{R}^n$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the symmetric positive-definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ is the vector of Coriolis and centripetal torques, $G_i(q_i) \in \mathbb{R}^n$ is the vector of gravitational torques, $\tau_i \in \mathbb{R}^n$ is the vector of control input generalized forces acting on the systems.

Throughout the subsequent analysis, we always assume that the leader's velocity and acceleration are accessible to followers, and the directed topology graph $\overline{\mathbb{G}}$ has a spanning tree. Consequently, the matrix H is positive stable and there exists a matrix $P > 0$ such that $HP + PH^T = Q > 0$ [14–16]. We also assume that the Lagrange systems have the following useful properties [14,19,20,28]:

Property 1 *For each agent i, there exist positive constants $k_{\overline{mi}}$, $k_{\underline{mi}}$, and k_{ci} such that $k_{\overline{mi}} \leq \|M_i(q_i)\| \leq k_{\underline{mi}}$ and $\|C_i(q_i, \dot{q}_i)\| \leq k_{ci}\|\dot{q}_i\|$.*

Property 2 *The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric, i.e., $\zeta^T[\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)]\zeta = 0$, for $\forall\zeta \in \mathbb{R}^n$.*

Property 3 *The left side of system (1) is linear in a set of constant physical parameters $\Theta_i = [\Theta_{i1}, \Theta_{i2}, \cdots, \Theta_{id}]^T \in \mathbb{R}^d$ as*

$$M_i(q_i)\dot{\xi}_i + C_i(q_i, \dot{q}_i)\xi_i + G_i(q_i) = Y_i(q_i, \dot{q}_i, \xi_i, \dot{\xi}_i)\Theta_i,$$

for all $\xi_i, \dot{\xi}_i \in \mathbb{R}^n$, where $Y_i(q_i, \dot{q}_i, \xi_i, \dot{\xi}_i)$ is called the dynamic regressor matrix which is a known matrix depending on the signals $q_i, \dot{q}_i, \xi_i, \dot{\xi}_i$, it is assumed that if the arguments of Y_i are bounded then Y_i is bounded. Θ_i is a vector for unknown but constant parameters associated with the ith agent.

For convenience, define $q = [q_1^T, q_2^T, \cdots, q_N^T]^T$, $v_0 = [\dot{q}_0^T, \dot{q}_0^T, \cdots, \dot{q}_0^T]^T \in \mathbb{R}^{nN}$, $M = \mathrm{diag}(M_1(q_1), M_2(q_2), \cdots, M_N(q_N))$, $C = \mathrm{diag}(C_1(q_1, \dot{q}_1), C_2(q_2, \dot{q}_2), \cdots, C_N(q_N, \dot{q}_N))$, $G = [G_1^T(q_1), G_2^T(q_2), \cdots, G_N^T(q_N)]^T$, $\tau = [\tau_1^T, \tau_2^T, \cdots, \tau_N^T]^T$.

3 Main results

In this section, based on backstepping approach, under the conditions that the leader's vector of position and velocity is varying, two cases of adaptive tracking control algorithms for Lagrange systems are presented. For the systems with uncertain external disturbance, the proposed algorithm can avoid online measurement and real-time processing for neighbors' velocities during control. Consequently, the online processing burden of control systems can be alleviated, and real-time can be more advanced. Furthermore the presented adaptive tracking control algorithm with external disturbance can overcome chattering, which may be caused by nonsmooth function sign. For the parameter uncertain systems, the proposed algorithm needs only subset of the followers who have access to leader's position.

3.1 Adaptive tracking control algorithm with external uncertain disturbance

In this subsection, we present a tracking adaptive algorithm that allows for uncertain external disturbance, which meet the practical constraints. Those uncertain external disturbances can be produced by uncertainty modeling, inertial friction and noise effect. What's more, a more effective and robust backstepping control algorithm that accounts for uncertain Lagrange systems is proposed to eliminate the chattering phenomena, which may be caused by nonsmooth function $\mathrm{sgn}(x)$. Consequently, the performance of the adaptive control algorithm is upgraded because of the smoothness of function $x^2\mathrm{sgn}(x)$, $x \in \mathbb{R}$.

Assume the system (1) with input disturbance, i.e., consider the following systems

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + f_i,\quad i = 1, 2, \cdots, N, \tag{2}$$

where $f_i = [f_{i1}, f_{i2}, \cdots, f_{in}]^T \in \mathbb{R}^n$ is bounded uncertain time-varying disturbance [19,20], which denotes external disturbance uncertainty modeling, internal friction, and noise effect of ith agent, and $|f_{ij}| \le f^*_{ij}$, $j = 1, 2, \cdots, n$, $f^*_i = [f^*_{i1}, f^*_{i2}, \cdots, f^*_{in}]^T \in \mathbb{R}^n$ is uncertain constant vector.

We consider a coordinated regulation algorithm as

$$\begin{aligned}\tau_i &= C_i\dot{q}_i + G_i + M_i\ddot{q}_0\\ &-M_i\beta_i\Big\{(K_{1i}+K_{2i})\sum_{j=1}^{N} a_{ij}\big[(q_i-q_j)+b_{i0}(q_i-q_0)\big]\\ &+(\dot{q}_i-\dot{q}_0)\Big\} - M_iK_{3i}\big[\eta_i\circ\eta_i\circ\mathrm{sgn}(\eta_i)\big]\\ &-\widehat{f}_i\circ\mathrm{sgn}(M_i^{-1}\eta_i),\end{aligned} \tag{3}$$

where K_{1i}, K_{2i}, K_{3i}, and $\beta_i \in \mathbb{R}^{n\times n}$ are candidate positive diagonal matrix, $\eta_i = (K_{1i} + K_{2i})\sum_{j=1}^{N} a_{ij}\big[(q_i-q_j)+b_{i0}(q_i-q_0)\big]+(\dot{q}_i-\dot{q}_0)$, $\widehat{f}_i$ is the estimate of f^*_i.

Let $A = K_1[(PH)\otimes I_n] + (H^T\otimes I_n)K_2[(K_1 + K_2)(H\otimes I_n)]K_1(H\otimes I_n)+(H^T\otimes I_n)K_2\beta K_2(H\otimes I_n)$, $B = -(P\otimes I_n) - (H^T\otimes I_n)K_2[(K_1 + K_2)(H\otimes I_n)]+ (H^T\otimes I_n)K_1 [(H^T\otimes I_n)(K_1+ K_2)]+ 2(H^T\otimes I_n)K_2\beta$, $D = \beta - (K_1 + K_2)(H\otimes I_n)$, where $K_1 = \mathrm{diag}(K_{11}, K_{12}, \cdots, K_{1N})$, $K_2 = \mathrm{diag}(K_{21}, K_{22}, \cdots, K_{2N})$, $K_3 = \mathrm{diag}(K_{31}, K_{32}, \cdots, K_{3N})$, $\beta = \mathrm{diag}(\beta_1, \beta_2, \cdots, \beta_N)$.

Let

$$\Phi = \begin{bmatrix} \frac{1}{2}(A+A^T) & \frac{1}{2}B\\ \frac{1}{2}B^T & \frac{1}{2}(D+D^T)\end{bmatrix}.$$

Theorem 1 *Using adaptive control algorithm (3) for the system (2), if at least one follower has access to q_0, and there exist positive-definite matrices K_1, K_2, K_3, and β, such that $\Phi > 0$, then $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$, $i = 1, 2, \cdots, N$, i.e., all followers can track the leader's dynamics.*

Proof Let $f = [f_1^T, f_2^T, \cdots, f_N^T]^T$, rewritten the system (2) as compact form

$$\ddot{q} = M^{-1}(-C\dot{q} - G + \tau + f). \tag{4}$$

First step: Let $\xi = q - 1_N\otimes q_0$, $v_0 = 1_N \otimes \dot{q}_0$, hence $\dot{\xi} = \dot{q} - v_0$, define the virtual variable $r = \dot{\xi} + K_1(H\otimes I_n)\xi$.

Consider Lyapunov function

$$V_1 = \frac{1}{2}\xi^T(P\otimes I_n)\xi. \tag{5}$$

The derivative of V_1 along the trajectory of (4) is

$$\begin{aligned}\dot{V}_1 &= \xi^T(P\otimes I_n)[r - K_1(H\otimes I_n)\xi]\\ &= \xi^T(P\otimes I_n)r - \xi^T[K_1((PH)\otimes I_n)]\xi.\end{aligned} \tag{6}$$

Second step: Consider Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\eta^T\eta + \frac{1}{2\gamma}\widetilde{f}^T\widetilde{f}, \tag{7}$$

where $\eta = K_2(H\otimes I_n)\xi + r$, $\widetilde{f} = f^* - \widehat{f}$, $f^* = [f_1^{*T}, f_2^{*T}, \cdots, f_N^{*T}]^T$, $\widehat{f}$ is the estimate of f^*.

Let $W = (M^{-1}\eta)\circ\,\mathrm{sgn}(M^{-1}\eta)$, $\varsigma^T = (\xi^T, r^T)$. Rewritten control input (3) as compact form $\tau = C\dot{q} + G + M\dot{v}_0 - M\beta[(K_1+K_2)(H\otimes I_n)\xi + \dot{\xi}] - MK_3\eta\circ\eta\circ\mathrm{sgn}(\eta) - \widehat{f}\circ\mathrm{sgn}(M^{-1}\eta)$.

The derivative of V_2 along the trajectory of (4) is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \eta^T\dot{\eta} - \frac{1}{\gamma}\widetilde{f}^T\dot{\widetilde{f}} = \dot{V}_1 + \eta^T[K_2(H\otimes I_n)\dot{\xi} + \dot{r}]\\ &\quad -\frac{1}{\gamma}\widetilde{f}^T\dot{\widehat{f}}\\ &= \dot{V}_1 + \eta^T[(K_1+K_2)(H\otimes I_n)\dot{\xi} + \ddot{q} - \dot{v}_0]\\ &\quad -\frac{1}{\gamma}\widetilde{f}^T\dot{\widehat{f}}\\ &= \dot{V}_1 + \eta^T[(K_1+K_2)(H\otimes I_n)\dot{\xi}\\ &\quad + M^{-1}(-C\dot{q} - G + \tau + f) - \dot{v}_0] - \frac{1}{\gamma}\widetilde{f}^T\dot{\widehat{f}}\\ &\le \dot{V}_1 + \eta^T\big[(K_1+K_2)(H\otimes I_n)\dot{\xi}\\ &\quad + M^{-1}(-C\dot{q} - G + \tau) - \dot{v}_0\big] + W^T\widehat{f}\\ &\quad -\frac{1}{\gamma}\widetilde{f}^T[\dot{\widehat{f}} - \gamma W].\end{aligned} \tag{8}$$

Choose adaptive updated law as

$$\dot{\widehat{f}} = \gamma W. \tag{9}$$

Thus

$$\begin{aligned}\dot{V}_2 &\leq \xi^T(P\otimes I_n)r - \xi^T[K_1((PH)\otimes I_n)]\xi + \eta^T[(K_1 \\ &\quad + K_2)(H\otimes I_n)\dot{\xi}] - \eta^T(\beta\eta + K_3\eta\circ\eta\circ \mathrm{sgn}(\eta)) \\ &= \xi^T(P\otimes I_n)r - \xi^T[K_1((PH)\otimes I_n)]\xi \\ &\quad + [\xi^T(H^T\otimes I_n)K_2 + r^T][(K_1+K_2)(H\otimes I_n)] \\ &\quad (r - K_1(H\otimes I_n)\xi)] \\ &\quad - [\xi^T(H^T\otimes I_n)K_2 + r^T]\beta[K_2(H\otimes I_n)\xi + r] \\ &\quad - K_3\eta^T(\eta\circ\eta\circ \mathrm{sgn}(\eta)) \\ &= -\varsigma^T\Phi\varsigma - K_3\eta^T(\eta\circ\eta\circ\mathrm{sgn}(\eta)). \end{aligned} \tag{10}$$

Note that $\Phi > 0$ and $-K_3\eta^T(\eta\circ\eta\circ\mathrm{sgn}(\eta)) \leq 0$, thus $\dot{V}_2 \leq 0$, by invariance-like theorem [26,29], $\|\varsigma\| \to 0$, i.e., $\|\xi\| \to 0$, $\|r\| \to 0$. Pay attention to $r = \dot{\xi} + (H\otimes I_n)\xi$, therefore $\|\dot{\xi}\| \to 0$, i.e., $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$.

Remark 1 Theorem 1 provide simple yet general criteria ensuring adaptive tracking in networked Lagrange systems subject to uncertain external disturbance in explicit expressions of a matrix. The condition $\Phi > 0$ is only sufficient but not necessary. In general, the well-known LMI tools of mathematical software such as MATLAB are quite useful to compute the feasible solution of the obtained criteria. Explicitly, if $\Phi > 0$ contains feasible solution, then the feasible solution can always be found out by the use of the LMI tools; thus, there exist control gains and $P > 0$, such that $\Phi > 0$, and thereby to yield an effective adaptive tracking scheme by Theorem 1, which leads to $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$ $(i = 1, 2, \cdots, N)$. This point will be further illustrated through Example 1 in Sect. 4 in detail.

In addition, if we choose $K_1 = K_2 = K_3 = k$ and β as positive constant scalars, the adaptive tracking control algorithm (3) can become a simple form

$$\begin{aligned}\tau_i &= C_i\dot{q}_i + G_i + M_i\ddot{q}_0 \\ &\quad - M_i\beta\left\{2k\sum_{j=1}^{N} a_{ij}\left[(q_i - q_j) + b_{i0}(q_i - q_0)\right]\right. \\ &\quad \left. + (\dot{q}_i - \dot{q}_0)\right\} - M_i k\left[\eta_i\circ\eta_i\circ\mathrm{sgn}(\eta_i)\right] \\ &\quad - \widehat{f}_i\circ\mathrm{sgn}(M^{-1}\eta), \end{aligned} \tag{11}$$

correspondingly, $A = k[(PH)\otimes I_n] + 2k^3[(H^TH^2)\otimes I_n] + k^2\beta[(H^TH)\otimes I_n]$, $B = -(P\otimes I_n) - 2k^2[(H^TH)\otimes I_n] + 2k^2[(H^T)^2\otimes I_n] + 2k\beta(H^T\otimes I_n)$, $D = \beta I_{nN} - 2k(H\otimes I_n)$; thus, the form of Φ is simple and practical.

Remark 2 For the system (2), the adaptive control input (3) can efficiently eliminate the chattering phenomena during tracking because of the smoothness of the term $\eta\circ\eta\circ\mathrm{sgn}(\eta)$, where $\mathrm{sgn}(\cdot)$ is defined componentwise, so the proposed control algorithm guarantees robustness with respect to uncertain disturbance. If we choose adaptive control input algorithm including the term $\mathrm{sgn}(\eta)$ which is similar to [16,21,23] as following

$$\begin{aligned}\tau &= C\dot{q} + g + M\dot{v}_0 - M\beta[(K_1 + K_2)(H\otimes I_n)\xi + \dot{\xi}] \\ &\quad - MK_3\mathrm{sgn}(\eta) - \widehat{f}\circ\mathrm{sgn}(M^{-1}\eta), \end{aligned} \tag{12}$$

accordingly, (10) can be written as follow

$$\dot{V}_2 \leq -\varsigma^T\Phi\varsigma - K_3\eta^T\mathrm{sgn}(\eta). \tag{13}$$

Note that $\Phi > 0$ and $-K_3\eta^T\mathrm{sgn}(\eta) \leq 0$; therefore, $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$. However, the discontinuous nonsmooth function $\mathrm{sgn}(\cdot)$ which is nondifferentiable and may cause chattering, which is usually undesirable in practice, since it involves high-frequency control logic switches and limit cycles. The system could even become unstable as a result of the chattering phenomena when un-modeled structure dynamics is excited [28].

Remark 3 The proposed control algorithm (3) shows that $\dot{q}_0$ and $\ddot{q}_0$ are accessible to every follower in the controller design phase. It is a common requirement in the existing literatures [17,18,26,27,30]. In contrast to the control algorithms requiring q_0 in [17,18,26,27,30], the designed control algorithm (3) illustrate that only a subgroup of the followers have access to leader's position q_0, and each follower can only get its neighbors' position information.

3.2 Adaptive tracking control algorithm with parametric uncertainties

We consider the following distribute leader adaptive tracking control algorithm for networked Lagrange systems with uncertain parameters. By backstepping approach, the appropriate Lyapunov function can be redesigned and selected carefully; consequently, the track control algorithm can be designed. Though the leader's velocity $\dot{q}_0$ and acceleration $\ddot{q}_0$ are accessible to every follower; in contrast to the existing algorithms in [17,18,26,27], the proposed control algorithm for the general networked Lagrange systems with uncertain parameters shows that only a subgroup of the followers have access to q_0.

Let $v_i^* = \dot{q}_0 - c_1\Big[\sum_{j=1}^{N} a_{ij}(q_i - q_j) + b_{i0}(q_i - q_0)\Big]$, where $c_1 > 0$ is feedback gain, from Property 3, we get that

$$M_i(q_i)\dot{v}_i^* + C_i(q_i,\dot{q}_i)v_i^* + G_i(q_i) = Y_i(q_i,\dot{q}_i,v_i^*,\dot{v}_i^*)\Theta_i, \quad i = 1,2,\cdots,N.$$

Since the value of the dynamic parameter Θ_i is hard to be known exactly in practice, one defines $\widehat{\Theta}_i$ is the estimate of Θ_i. Let $v_i = \dot{q}_i$, $z_i = v_i - v_i^*$.

Consider the adaptive coordinated regulation algorithm as

$$\tau_i = \widehat{M}_i(q_i)\dot{v}_i^* + \widehat{C}_i(q_i,\dot{q}_i)v_i^* + \widehat{G}_i(q_i) - z_i = Y_i(q_i,\dot{q}_i,v_i^*,\dot{v}_i^*)\widehat{\Theta}_i - z_i, \tag{14}$$

where $\widehat{M}_i(q_i)$, $\widehat{C}_i(q_i,\dot{q}_i)$, and $\widehat{G}_i(q_i)$ represent the estimates of the matrices available at that instant.

The estimated parameter Θ_i is updated by the following adaption law

$$\dot{\widehat{\Theta}}_i = -\Gamma_i Y_i^T z_i, \tag{15}$$

where Γ_i is a symmetric positive-definite matrix, $i = 1,2,\cdots,N$.

Theorem 2 *Using adaptive control algorithm (14) for the system (1), if at least one follower has access to q_0, then there exists an appropriate control gain $c_1 > 0$, such that $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$, $i = 1,2,\cdots,N$, i.e., all followers can track the leader's dynamics.*

Proof For convenience, let $v^* = [v_1^{*T}, v_2^{*T},\cdots, v_N^{*T}]^T$, $v = [v_1^T, v_2^T,\cdots,v_N^T]^T$, $z = [z_1^T, z_2^T,\cdots, z_N^T]^T$, $Y = \mathrm{diag}(Y_1,Y_2,\cdots,Y_N)$, $\Theta = [\Theta_1^T, \Theta_2^T,\cdots, \Theta_N^T]^T$, $\widehat{\Theta} = [\widehat{\Theta}_1^T, \widehat{\Theta}_2^T,\cdots,\widehat{\Theta}_N^T]^T$, $\widetilde{\Theta} = \Theta - \widehat{\Theta}$, where $\widetilde{\Theta}$ denotes the estimation error, clearly $\dot{\widetilde{\Theta}} = -\dot{\widehat{\Theta}}$.

Two steps of backstepping approach are proceed to find the real control input algorithm.

First step: The virtual control v_i^* for q_i can be written as the compact form $v^* = -c_1(H \otimes I_n)\xi + v_0$. By $z = v - v^*$, the system (1) can be rewritten as compact form $M\dot{z} + M\dot{v}^* + Cz + Cv^* + G = Y\widehat{\Theta} - z$, i.e.,

$$M\dot{z} + Cz = -Y\widetilde{\Theta} - z. \tag{16}$$

For the following system

$$\begin{cases} \dot{q} = z + v^*, \\ \dot{z} = -M^{-1}Cz - M^{-1}Y\widetilde{\Theta} - M^{-1}z. \end{cases} \tag{17}$$

Consider the following candidate Lyapunov function

$$V_3 = \frac{1}{2}\xi^T(P \otimes I_n)\xi. \tag{18}$$

By $\xi = q - 1_N \otimes q_0$, we can get $\dot{\xi} = v - v_0$, thus

$$\begin{aligned}\dot{V}_3 &= \xi^T(P \otimes I_n)\dot{\xi} = \xi^T(P \otimes I_n)(z + v^* - v_0) \\ &= \xi^T(P \otimes I_n)z + \xi^T(P \otimes I_n)(-c_1(H \otimes I_n)\xi) \\ &= \xi^T(P \otimes I_n)z - c_1\xi^T[(PH) \otimes I_n]\xi \\ &= \xi^T(P \otimes I_n)z - \frac{1}{2}c_1\xi^T(Q \otimes I_n)\xi, \end{aligned} \tag{19}$$

the first term of $\dot{V}_1$ will be dealt in the next step of the backstepping procedure.

Second step: Consider new Lyapunov function as

$$V_4 = V_3 + \frac{c_2}{2}z^TMz + \frac{1}{2}\widetilde{\Theta}^T\Xi\widetilde{\Theta}, \tag{20}$$

where $c_2 > 0$ is candidate constant, $\Xi = \Gamma^{-1} = \mathrm{diag}(\Gamma_1^{-1},\Gamma_2^{-1},\cdots,\Gamma_N^{-1})$.

Let $\lambda_1(\cdot)$ and $\lambda_N(\cdot)$ denote minimal and maximal eigenvalues of matrix. Rewritten control input algorithm (14) as compact form $\tau = Y\widehat{\Theta} - z$, according to Property 2 and adaptive updated law (15), derivative V_4 along the trajectory of (17) with respect to time, we have

$$\begin{aligned}\dot{V}_4 &= \dot{V}_3 + c_2z^TM\dot{z} + \frac{c_2}{2}z^T\dot{M}z \\ &= \dot{V}_3 + \frac{c_2}{2}z^T(\dot{M} - 2C)z + c_2z^TM[-M^{-1}Cz \\ &\quad -M^{-1}Y\widetilde{\Theta} - M^{-1}z] + c_2z^TCz + \widetilde{\Theta}^T\Xi\dot{\widetilde{\Theta}} \\ &= \dot{V}_3 - c_2z^TY\widetilde{\Theta} - c_2z^Tz - \widetilde{\Theta}\Xi\dot{\widehat{\Theta}} \\ &= \dot{V}_3 - c_2z^TY\widetilde{\Theta} + c_2\widetilde{\Theta}^TY^Tz - c_2z^Tz \\ &= \xi^T(P \otimes I_n)z - \frac{1}{2}c_1\xi^T(Q \otimes I_n)\xi - c_2z^Tz \\ &\le -\frac{c_1}{2}\xi^T(Q \otimes I_n)\xi + \frac{1}{2\varepsilon}\xi^T(P^2 \otimes I_n)\xi \\ &\quad +\frac{\varepsilon}{2}z^Tz - c_2z^Tz \\ &= -\xi^T\Big[\frac{c_1}{2}(Q \otimes I_n) - \frac{1}{2\varepsilon}(P^2 \otimes I_n)\Big]\xi \\ &\quad -z^T\Big(c_2 - \frac{\varepsilon}{2}\Big)z \\ &\le -\Big(\frac{c_1\lambda_1(Q)}{2} - \frac{\lambda_N(P^2)}{2\varepsilon}\Big)\|\xi\|_2^2 - \Big(c_2 - \frac{\varepsilon}{2}\Big)\|z\|_2^2. \end{aligned} \tag{21}$$

Choose suitable positive parameters c_1, c_2, and ε, such that $c_1 > \frac{\lambda_N(P^2)}{\varepsilon\lambda_1(Q)}$ and $c_2 > \frac{\varepsilon}{2}$, thus $\dot{V}_4 \le 0$. Similar to Theorem 1, $\|\xi\| \to 0$ and $\|z\| \to 0$. From $z = v - v^* = \dot{q} + c_1(H \otimes I_n)\xi - v_0$, we conclude that $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$, as $t \to \infty$.

4 Application examples

In this section, we shall discuss the applications of the above theoretic criteria. Two examples and their simulations are given to show that our main results are practical. The simulations are performed with a network of four nonidentical two-link revolute joint manipulators which track a time-varying leader, where each manipulator dynamics is described by the Lagrange systems. As discussed in Sect. 3, the topology graph is directed, with a spanning tree and at least one follower can access to q_0, (see Fig. 1).

Example 1 Consider four two-link revolute nonidentical manipulators with directed graph:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + f_i,$$
$$i = 1,\ 2,\ 3,\ 4, q_i \in \mathbb{R}^2. \tag{22}$$

where the inertia matrices, Coriolis and centrifugal matrices, and the gravity vectors are given by

$$M_i(q_i) = \begin{bmatrix} \Theta_{i1} + 2\Theta_{i2}\cos(q_{i2}) & \Theta_{i3} + \Theta_{i2}\cos(q_{i2}) \\ \Theta_{i3} + \Theta_{i2}\cos(q_{i2}) & \Theta_{i3} \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} -\Theta_{i2}\sin(q_{i2})\dot{q}_{i2} & -\Theta_{i2}\dot{q}_{i12}\sin(q_{i2}) \\ \Theta_{i2}\sin(q_{i2})\dot{q}_{i1} & 0 \end{bmatrix},$$

$$G_i(q_i) = \begin{bmatrix} \Theta_{i4}g\cos(q_{i1}) + \Theta_{i5}g\cos(q_{i12}) \\ \Theta_{i5}g\cos(q_{i12}) \end{bmatrix},$$

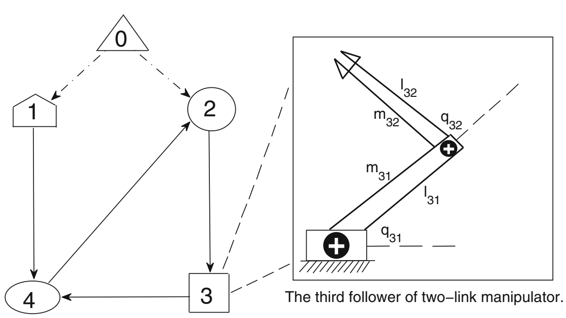

Fig. 1 The position communication topology graph of the leader and four followers nonidentical two-link manipulators

$$\begin{bmatrix} \Theta_{i1} \\ \Theta_{i2} \\ \Theta_{i3} \\ \Theta_{i4} \\ \Theta_{i5} \end{bmatrix} = \begin{bmatrix} m_{i1}l_{c_{i1}}^2 + m_{i2}(l_{i1}^2 + l_{c_{i2}}^2) + J_{i1} + J_{i2} \\ m_{i2}l_{i1}l_{c_{i2}} \\ m_{i2}l_{c_{i2}}^2 + J_{i2} \\ m_{i1}l_{c_{i1}} + m_{i2}l_{i1} \\ m_{i2}l_{c_{i2}} \end{bmatrix},$$

where q_{i12} and $\dot{q}_{i12}$ denote $q_{i1} + q_{i2}$ and $\dot{q}_{i1} + \dot{q}_{i2}$, respectively, $l_{c_{i1}} = \frac{1}{2}l_{i1}$, $l_{c_{i2}} = \frac{1}{2}l_{i2}$, $g = 9.8m/s^2$ is the acceleration of gravity constant. For the ith manipulator, m_{i1} and m_{i2} are the masses of links 1 and 2, respectively; l_{i1} and l_{i2} are the respective lengths of links 1 and 2, (see Fig. 1). The moment of inertia of links 1 and 2 are $J_{i1} = \frac{1}{3}m_{i1}l_{c_{i1}}^2$ and $J_{i2} = \frac{1}{3}m_{i2}l_{c_{i2}}^2$. q_{i1} and q_{2i} are the articular positions of links 1 and 2, respectively. The adaptive tracking control input vector τ_i is the generalized forces and moments acting on systems.

Obviously, matrix $M_i(q_i)$ is symmetric, positive definite, differentiable, and satisfies the Property 1 and 2. $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $G_i(q_i)$ have inherent nonlinearity. Assume that the system (22) is using feedback control input (3).

The agents are assumed to be communicated using directed topology graph $\overline{\mathbb{G}}$ as shown in Fig. 1. An arrow from node i to node j indicates that agent j can obtain position information from agent i. Matrix H can be shown as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

The vector of joint angles of the leader is chosen as $q_0(t) = [0.21\sin(0.82t),\ 0.2 + 0.26\cos(0.5t)]^T$ rad, hence vector of the leader's velocity $\dot{q}_0(t)$ and accelerate velocity $\ddot{q}_0(t)$ can be computed easily. Choose the time-varying disturbance as $f_i = 0.6[0.25\sin(0.9t + 0.02i), 0.5\cos(0.6t + 0.03i)]^T$. The initial value can be randomly chosen as $[q_1^T(0), \dot{q}_1^T(0), \hat{f}_1^T(0), q_2^T(0), \dot{q}_2^T(0), \hat{f}_2^T(0), \cdots, q_4^T(0), \dot{q}_4^T(0), \hat{f}_4^T(0)]^T = [-1.7327, 1.4017, -1.2887, -1.8803, 0.9257, 0.1361, -0.9059, -0.5362, -1.9585, 1.5468, 1.4541, -0.9930, 0.2679, -1.3729, 0.3675, -0.6856, 0.6245, 1.4445, 0.2605, 1.9119, 1.1573, -1.3996, 1.3221, -1.2425]^T$ in $(-2.01, 1.9)$, where $q_i(0), \dot{q}_i(0)$ and $\widehat{f}_i(0) \in \mathbb{R}^2, i = 1,\ 2,\ 3,\ 4$.

Let $m_{i1} = 1.2 + 0.2i$, $m_{i2} = 1.4 + 0.12i$, $l_{i1} = 1.8 + 0.08i$, $l_{i2} = 2.3 + 0.04i$, $i = 1, 2, 3, 4$, $g = 9.8m/s^2$, $\gamma = 0.8$, clearly, the lengths and masses of

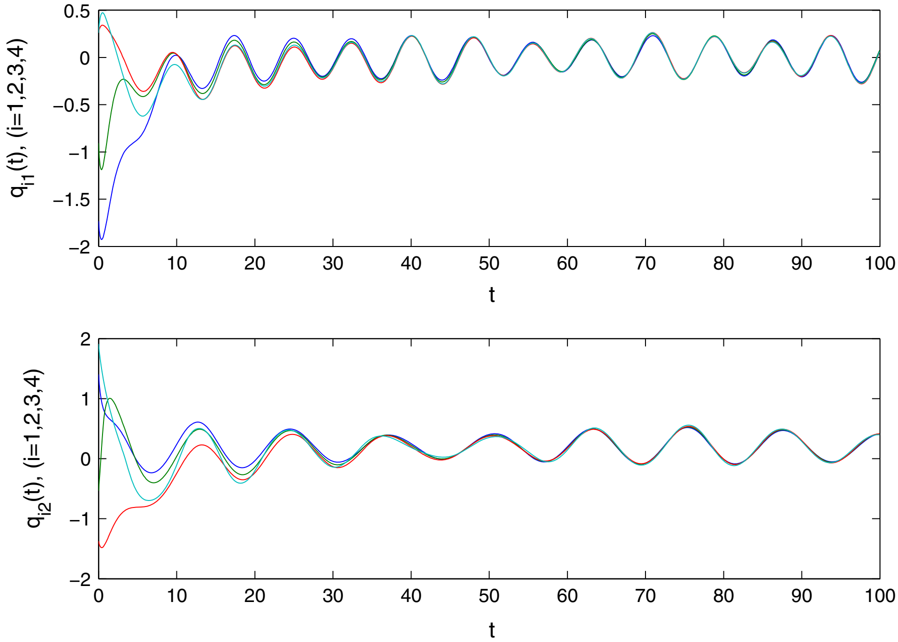

Fig. 2 The joint angle tracking results of the system (22) for leader joint angle $q_0(t)$ with input control (3)

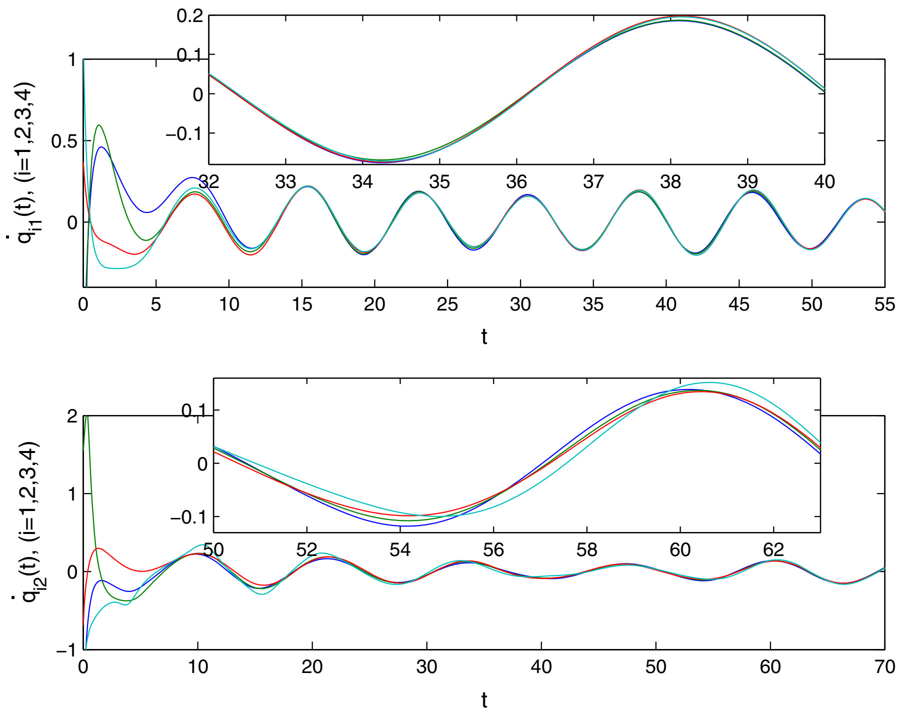

Fig. 3 The joint angle velocity tracking results of the system (22) for leader joint angle velocity $\dot{q}_0(t)$ with input control (3)

each agent are different, i.e., the network systems are nonidentical.

Let $K_1 = 0.08(\text{diag}(1.4, 0.76) \otimes I_4)$, $K_2 = 0.12(\text{diag}(0.98, 0.86) \otimes I_4)$, $\beta = 2.5$ (diag(0.97, 1.2)$\otimes I_4$), by LMI tools in MATLAB 7.0 platform, there exists feasible solution $P = [P_1, P_2, P_3, P_4] \in \mathbb{R}^{4\times 4}$, such that $\Phi > 0$ and $Q > 0$, where $P_1 = [1.2487, -0.1094, -0.0097, -0.2822]^T$,

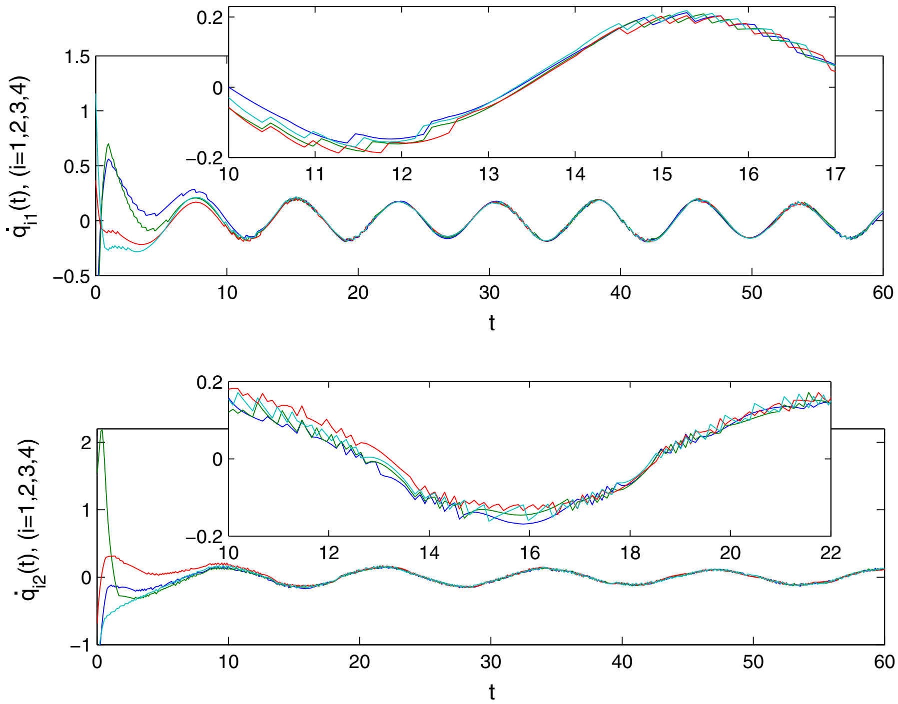

Fig. 4 The joint angle velocity tracking results of the system (22) for leader joint angle velocity $\dot{q}_0(t)$ with input control (12)

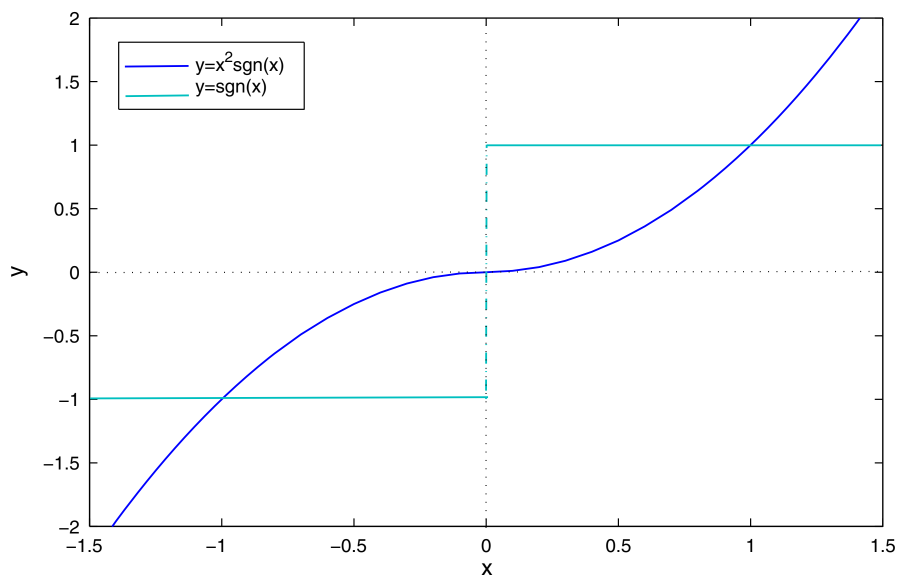

Fig. 5 Comparison of curves of $y = \mathrm{sgn}(x)$ and $y = x^2\mathrm{sgn}(x)$

$P_2 = (-0.1094,\ 1.5542,\ -0.2964,\ -0.1155)^T$, $P_3 = [-0.0097,\ -0.2964,\ 1.1582,\ -0.3607]^T$, $P_4 = [-0.2822,\ -0.1155,\ -0.3607,\ 1.6041]^T$.

In fact, the minim eigenvalue of Φ is $\lambda_1(\Phi) = 0.0061 > 0$. Hence the condition of Theorem 1 is satisfied, thus $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$, $i = 1, 2, 3, 4$, i.e., all followers' joint angles and velocities approach those of the leader, (see Figs. 2, 3).

However, if we choose adaptive control input algorithm (12), the chattering phenomena appears during tracking clearly, (see Fig. 4), where all parameters are the same as Fig. 3. The comparison of smooth curve

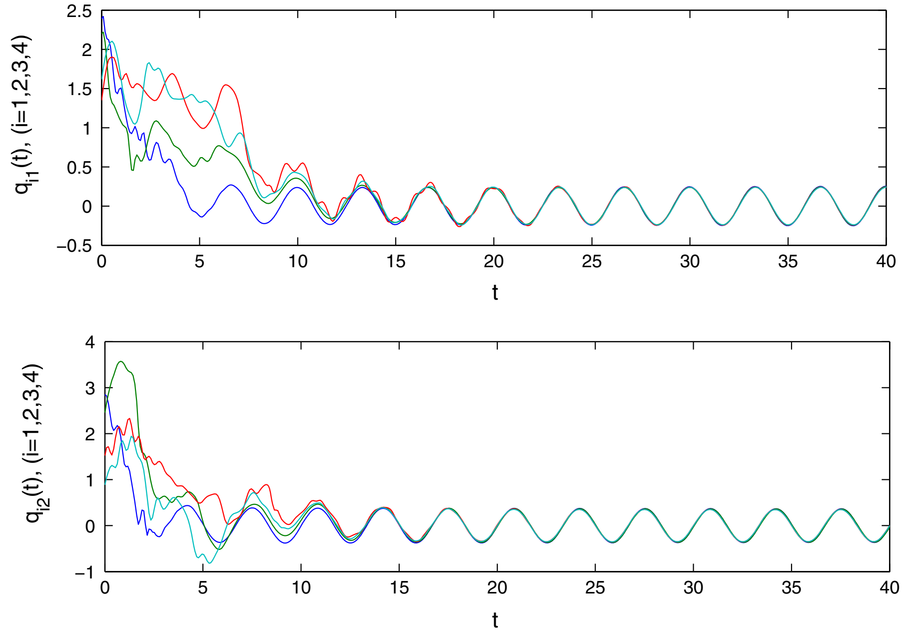

Fig. 6 Joint angle tracking results of the system (23) for leader's joint angle $q_0(t)$ with input control (14)

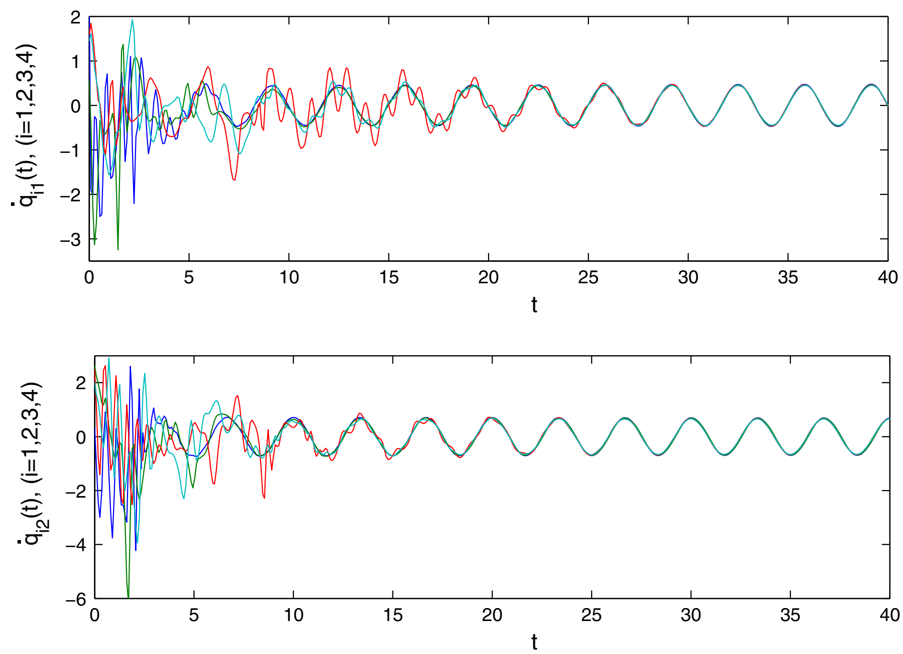

Fig. 7 Joint angle velocities tracking results of system the (23) for leader's joint angle velocity $\dot{q}_0(t)$ with input control (14)

$x^2\mathrm{sgn}(x)$ and non-smooth curve $\mathrm{sgn}(x)$ is shown Fig. 5, $x \in \mathbb{R}$.

Example 2 Consider four two-link revolute nonidentical manipulators with directed topology graph, each manipulator nonlinear dynamics follows the Lagrange systems:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i,$$
$$i = 1,\ 2,\ 3,\ 4,$$
$$q_i = [q_{i1},\ q_{i2}]^T \in \mathbb{R}^2, \tag{23}$$

The adaptive tracking control input τ_i is chosen as (14). The agents are assumed to be communicated using

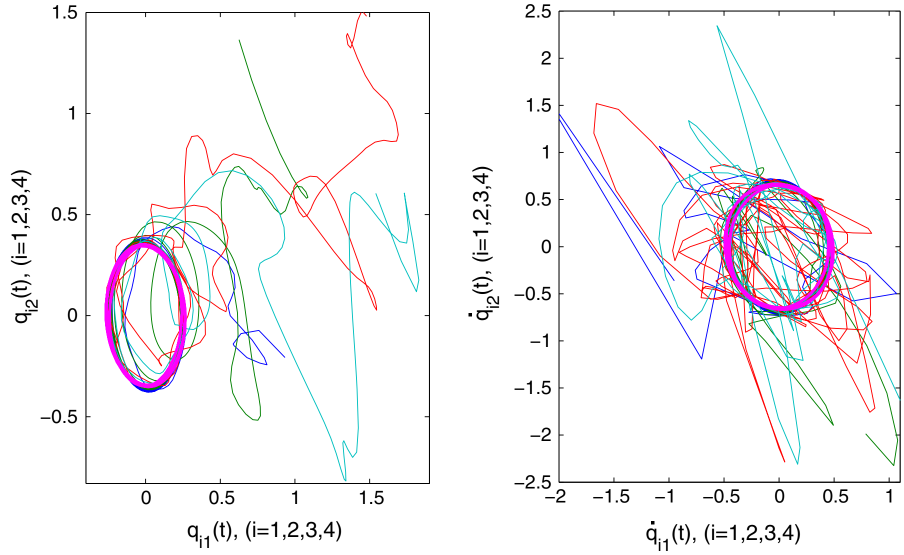

Fig. 8 Joint angle and velocity phase portraits of adaptive tracking controlled uncertain system (23), $t \in [2.25, 80]s$. For circular portraits of *purple*, $t \in [67.5, 80]s$

directed topology graph $\overline{\mathbb{G}}$, and matrix H is the same as Example 1.

The dynamic regressor matrix is as following:

$q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$, $i = 1, 2, 3, 4$, i.e., all followers' joint angles and velocities approach those of the leader, (see Figs. 6, 7, 8).

$$Y_i = \begin{bmatrix} \dot{v}_{i1}^* & 0 \\ (2\dot{v}_{i1}^* + \dot{v}_{i2}^*)\cos(q_{i2}) - (v_{i1}^*\dot{q}_{i2} + \dot{v}_{i2}^*\dot{q}_{i12})\sin(q_{i2}) & \dot{v}_{i1}^*\cos(q_{i2}) + v_{i1}^*\dot{q}_{i1}\sin(q_{i2}) \\ \dot{v}_{i2}^* & \dot{v}_{i1}^* + \dot{v}_{i2}^* \\ g\cos(q_{i1}) & 0 \\ g\cos(q_{i12}) & g\cos(q_{i12}) \end{bmatrix}^T.$$

The vector of joint angles of the leader is chosen as $q_0(t) = [0.25\cos(0.6\pi t), 0.375\sin(0.6\pi t)]^T$ rad, hence the vector of leader's velocity $\dot{q}_0(t)$ and accelerate velocity $\ddot{q}_0(t)$ can be computed easily. The initial value can be randomly chosen as $[q_1^T(0), \dot{q}_1^T(0), \widehat{\Theta}_1^T(0), q_2^T(0), \dot{q}_2^T(0), \widehat{\Theta}_2^T(0), \cdots, q_4^T(0), \dot{q}_4^T(0), \widehat{\Theta}_4^T(0)]^T = [2.3895,\ 2.8473,\ 2.5136,\ 0.3347,\ 1.0456,\ 0.7612,\ 0.9899,\ 2.8565,\ 1.6351,\ 2.1979,\ 2.4572,\ 0.5673,\ 2.6287,\ 0.3943,\ 2.1863,\ 2.1422,\ 2.1089,\ 0.4736,\ 1.3482,\ 1.5246,\ 1.5854,\ 2.5072,\ 1.9976,\ 2.3563, 1.5875,\ 2.7765,\ 0.2867,\ 1.6163,\ 0.8886,\ 1.4465,\ 2.0176, 0.6831,\ 1.8029,\ 1.0133,\ 2.5664,\ 0.4735]^T$ in $(0.1, 1.5)$, where $q_i(0) \in \mathbb{R}^2, \dot{q}_i(0) \in \mathbb{R}^2, \widehat{\Theta}_i(0) \in \mathbb{R}^5, i = 1, 2, 3, 4$.

Let $P = 0.23I_4$, $\varepsilon = 0.98$, $\Gamma = 8.5I_{20}$, $m_{i1} = 1.2 + 0.31i$, $m_{i2} = 1.1 + 0.25i$, $l_{i1} = 1.3 + 0.33i$, $l_{i2} = 2.5 + 0.15i$, $i = 1, 2, 3, 4$. If we choose $c_1 = 0.73$, $c_2 = 0.56$, such that $c_1 > \frac{\lambda_N(P^2)}{\varepsilon\lambda_1(Q)} = 0.2846$ and $c_2 > \frac{\varepsilon}{2} = 0.49$, thus the conditions of Theorem 2 are satisfied,

5 Conclusions

This paper presents two tracking control algorithms of general nonidentical networked Lagrange systems using backstepping scheme and directed graph theory. The proposed algorithms work to track time-varying position and velocity vectors of the leader. Moreover, the present adaptive tracking control algorithm with external disturbance is without measuring the neighbors' velocities. Furthermore, the tracking adaptive algorithm for Lagrange systems with uncertain disturbance is robust, and it can eliminate chattering in the recursive backstepping control may be caused by discontinuous function sign. For the parameter uncertain systems, the proposed adaptive tracking control algorithm only require a subset of the followers access to leader's position. The effectiveness of the proposed control algorithms is examined by two examples and simulations for Lagrange systems.

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